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INVESTIGATION OF INITIAL STRESS ON TORSIONAL VIBRATIONS IN AN ANISOTROPIC MAGNETO-POROELASTIC HOLLOW CYLINDERS

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ABSTRACT

In the present paper, torsional waves in an anisotropic magneto-poroelastic hollow cylinder are studied in the presence of initial stress. Governing equations are derived from Biot's theory of deformation. The frequency equation is obtained with the help of boundaries conditions. Frequency against the ratio of thickness to inner radius for different values of initial stress is calculated. The result obtained theoretically is computed for two types of materials and are presented graphically.

Keywords: Anisotropic, Poroelasticity, Magnetic field, Frequency equation, Frequency, Hollow cylinder, Torsional vibrations, Initial stress.

INTRODUCTION

The study of torsional vibrations in anisotropic is important in geoengineering applications. Propagation of torsional surface wave in anisotropic poroelastic medium under initial stress is investigated in [1]. Response of an anisotropic liquid saturated porous medium due to two dimensional sources is presented in [2]. Wave propagation in anisotropic liquid saturated porous solids is discussed in [3]. Magnetoelastic torsional waves in a bar under initial stress are explored in [4]. Propagation of shear wave in anisotropic medium is investigated in [5]. Effect of initial stress and magnetic field on propagation of shear wave in non homogeneous anisotropic medium under gravity field is investigated in [6]. Dispersion of torsional surface waves in anisotropic layer over porous half space under gravity is presented in [7]. Plane wave propagation in a rotating anisotropic medium with voids under the action of a uniform magnetic field is discussed in [8]. Effect of irregularity on torsional surface waves in an initially stressed anisotropic porous layer sandwiched between homogeneous and non homogeneous half space is presented in [9]. Longitudinal and torsional shock waves in anisotropic elastic cylinders are investigated in [10]. Torsional vibrations on anisotropic half space are studied in [11].The torsional surface wave in a prestressed anisotropic intermediate poroelastic layer of varying heterogeneities is discussed in [12]. Torsional waves in fluid saturated porous layer clamped between two anisotropic media are investigated in [13]. Torsional wave in dissipative cylindrical shell under initial stresses is presented in [14]. Torsional surface wave in gravitating anisotropic porous half space is discussed in [15]. In all the above papers we cannot find the magnetic field and initial stress in anisotropic poroelastic hollow cylinders.

The present paper attempts to study the propagation of torsional vibrations in an anisotropic magneto-poroelastic hollow cylinder in the presence of initial stress. It is observed that the presence of a magnetic field and initial stress always allow the torsional surface wave to propagate. The frequency equation is obtained in the presence of magnetic field and initial stress.

2. Governing equations and solution of the problem

Let $(\mathrm{r},\theta,\mathrm{z})$ be the cylindrical polar coordinates with inner and outer radii a and b, respectively, whose axis is in the direction of z -axis. The dynamic equations of motion of an anisotropic porous medium under the effect of initial stress and body forces are [16].

$$\frac{\partial\tau_{rr}}{\partial r}+\frac{1}{r}\frac{\partial\tau_{r\theta}}{\partial\theta}+\frac{\partial\tau_{rz}}{\partial z}+\frac{\tau_{rr}-\tau_{\theta\theta}}{r}-P\frac{\partial\omega_{\theta}}{\partial z}+F_r=\frac{\partial^2}{\partial t^2}(\rho_{11}u+\rho_{12}U),$$
$$\frac{\partial\tau_{\theta r}}{\partial r}+\frac{1}{r}\frac{\partial\tau_{\theta\theta}}{\partial\theta}+\frac{\partial\tau_{\theta z}}{\partial z}+\frac{2\tau_{r\theta}}{r}+P\frac{\partial\omega_{\theta}}{\partial z}+F_{\theta}=\frac{\partial^2}{\partial t^2}(\rho_{11}v+\rho_{12}V),$$
$$\frac{\partial\tau_{zr}}{\partial r}+\frac{1}{r}\frac{\partial\tau_{z\theta}}{\partial\theta}+\frac{\partial\tau'_{zz}}{\partial z}+\frac{\tau_{rz}}{r}-P(\frac{\partial\omega_{\theta}}{\partial r}-\frac{\partial\omega_r}{\partial\theta})+F_z=\frac{\partial^2}{\partial t^2}(\rho_{11}w+\rho_{12}W),$$
$$\frac{\partial\tau}{\partial r}=\frac{\partial^2}{\partial t^2}(\rho_{12}u+\rho_{22}U),$$
$$\frac{\partial\tau}{\partial\theta}=\frac{\partial^2}{\partial t^2}(\rho_{12}v+\rho_{22}V),$$
$$\frac{\partial\tau}{\partial z}=\frac{\partial^2}{\partial t^2}(\rho_{12}w+\rho_{22}W). \tag{1}$$

Where $\vec{u}(u,v,w)$ and $\vec{U}(U,V,W)$ be the solid and fluid displacements. ρ_{ij} are mass coefficients. $\tau_{rr},\tau_{\theta\theta},\tau_{zz},\tau_{rz},\tau_{r\theta},\tau_{\theta z}$ are stresses components and ω_r,ω_θ are rotational components, fluid pressure s . $F=(F_r,F_\theta,F_z)$ is the Lorentz force per unit volume due to the axial magnetic field is given by [4]

$$F=J\times B,\quad \omega_r=\frac{1}{2}(\frac{\partial w}{\partial\theta}-\frac{\partial v}{\partial\theta}),\quad \omega_r=\frac{1}{2}(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial r}) \tag{2}$$

The stress components for porous medium are given by

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$$\tau_{rr} = (A+P)e_{rr} + (A-2N+P)e_{\theta\theta} + (F+P)e_{zz} + Q_{\varepsilon},$$
$$\tau_{\theta\theta} = (A-2N)e_{rr} + Ae_{\theta\theta} + Fe_{zz} + Q_{\varepsilon},$$
$$\tau_{zz} = Fe_{rr} + Fe_{\theta\theta} + Ce_{zz} + Q_{\varepsilon},$$
$$\tau_{r\theta} = 2Ne_{r\theta},\ \tau_{\theta z} = 2Ge_{r\theta},\ \tau_{rz} = 2Ge_{zr},$$

(3)

Where A, F, C are elastic constants of the medium, N and G are the shear moduli along r, z directions respectively and Q_{ε} is the measure of the coupling between the volume of the solid and liquid. The relation between stress vector τ and the fluid pressure P' is $-\tau = fP'$. f is the porosity of the layer. The mass coefficients $\rho_{11}, \rho_{12}, \rho_{22}$ are related to the densities ρ, ρ_s, ρ_w of the layer, the solid and water respectively by $\rho_{11} + \rho_{12} = (1-f)\rho_s, \rho_{12} + \rho_{22} = f\rho_w$. Hence the mass density aggregate is $\rho' = \rho_s + f(\rho_w - \rho_s)$. Biot has shown that the following inequalities hold for the mass coefficients $\rho_{11} > 0, \rho_{22} > 0, \rho_{12} < 0, \rho_{11}\rho_{22} - \rho_{12}^2 > 0$. Maxwell equations governing the electromagnetic fields for slowly moving solid medium having electrical conductivity are [4]

$$curlH = 4\pi J, \quad curlE = \frac{-1}{c}\frac{\partial B}{\partial t}, \quad divB = 0, \quad B = \mu_e H$$

(4)

Where displacement current is neglected and by ohm's law

$$J = \sigma\left(E + \frac{1}{c}\frac{\partial u}{\partial t} \times B\right)$$

(5)

In eq. (4) and (5) H, B, E, J are respectively the magnetic intensity, magnetic induction, electric intensity and current density vectors; μ_e, σ, u are magnetic permeability and electrical conductivity of the body, displacement vector in strained state and c is the velocity of light. Now let suppose that $H = H_0 + h$, where H_0 the initial magnetic field acting parallel to is h is small perturbation in the field. If the cylinder is a perfect conductor of electricity (i.e, $\sigma \to \infty$) hence eq. (5) gives

$$E = \frac{-1}{c}\frac{\partial u}{\partial t} \times B = \left(\frac{-H}{c}\frac{\partial v}{\partial t}, 0, 0\right)$$

(6)

Where $H = |H_0|$. Eliminating E from eq. (4) and eq. (6) we obtain

$$h = \left(0,\ H\frac{\partial v}{\partial z},\ 0\right)$$

(7)

From eq. (4) and eq. (7) we get

$$J \times B = J \times \mu_e H = (0, \frac{-H^2}{4\pi}\frac{\partial^2 v}{\partial z^2}, 0)$$

(8)

In the case of torsional vibrations, the equations of motion eq. (1) reduced to the following equations:

$$\frac{\partial \tau_{\theta r}}{\partial r} + \frac{\partial \tau_{\theta z}}{\partial z} + \frac{2\tau_{r\theta}}{r} - (\frac{P}{2} - \frac{H^2}{4\pi})\frac{\partial^2 v}{\partial z^2} = \frac{\partial^2}{\partial t^2}(\rho_{11} v + \rho_{12} V),$$
$$0 = \frac{\partial^2}{\partial t^2}(\rho_{12} v + \rho_{22} V).$$

(9)

Assume that the wave solution takes the following form

$$v(r) = v_1(r) e^{ik(z-\omega t)},$$
$$V(r) = V_1(r) e^{ik(z-\omega t)}.$$

(10)

In eq. (10) ω is the frequency, k is the wavenumber, and t is time. Substituting eq. (10) and eq. (3) in eq. (9), we obtains

$$N\frac{d^2 v_1}{dr^2} + \frac{N}{r}\frac{dv_1}{dr} - N\frac{v_1}{r^2} - k^2(G - \frac{P}{2} - \frac{H^2}{4\pi})v_1 = -\omega^2(\rho_{11} v_1 + \rho_{12} V_1),$$
$$0 = -\omega^2(\rho_{12} v_1 + \rho_{22} V_1)$$

(11)

The solution of eq. (11) takes the following form

$$v_1(r) = [AJ_1(qr) + BY_1(qr)]e^{ik(z-\omega t)},$$
$$V_1(r) = \frac{-\rho_{11}}{\rho_{22}}[AJ_1(qr) + BY_1(qr)]e^{ik(z-\omega t)}.$$

(12)

Where $q = \frac{\omega^2}{V_s^2} - k^2(\frac{1}{N/G} - \frac{P}{2N} - \frac{H^2}{4\pi N})$. A, B are the arbitrary constants and $J_1(qr), Y_1(qr)$ are Bessel's functions of first and second kind. V_s is the shear wave velocity. The non zero stresses are

$$\sigma_{r\theta} = A[NqJ_0(qr) - \frac{2N}{r}J_1(qr)] + B[NqY_0(qr) - \frac{2N}{r}Y_1(qr)].$$

(13)

3. Boundary conditions and frequency equation

The boundary conditions which specify the inner and outer surface are free at $r=a$ and $r=b$ are

$$\sigma_{r\theta}=0 \text{ at } r=a \text{ and}$$
$$\sigma_{r\theta}=0 \text{ at } r=b \tag{14}$$

Using eqs. (13) and (14), we obtain two homogeneous equations

$$A[NqJ_0(qa)-\frac{2N}{a}J_1(qa)]+B[NqY_0(qa)-\frac{2N}{a}Y_1(qa)]=0,$$
$$A[NqJ_0(qb)-\frac{2N}{b}J_1(qb)]+B[NqY_0(qb)-\frac{2N}{b}Y_1(qb)]=0. \tag{15}$$

Eliminating the constants we obtain frequency equation

$$\begin{vmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{vmatrix}=0 \tag{16}$$

4. Numerical results

The frequency equation eq. (16), is used to calculate the numerical results. For two different types of cylinders, frequency is computed. Sandstone saturated with water makes up cylinders I [17] and cylinder-II, sandstone saturated with kerosene [18]. For different initial stress, one can establish an implicit relationship between frequency and the ratio of thickness to inner radius by using the values from the frequency equation. Let $g=\frac{b}{a}$ so that $\frac{h}{a}=g-1$. In Table 1, the physical characteristics of cylinders are listed. The mgneto-poroelastic coupling factor $\frac{H^2}{N}=0.01$ are taken from the thesis [19] and $\frac{N}{G}=2$ is given in [9]

Table-1

Cylinder-I	Cylinder-II
$N=0.2765\times10^{10}\,N/m^2$ $\rho_{11}=1.926137\times10^3\,kg/m^3$ $\rho_{12}=-0.00213\times10^3\,kg/m^3$ $\rho_{22}=0.21537\times10^3\,kg/m^3$	$N=0.922\times10^{10}\,N/m^2$ $\rho_{11}=1.90302\times10^3\,kg/m^3$ $\rho_{12}=0$ $\rho_{22}=0.268\times10^3\,kg/m^3$

Figure 1 depicts the frequency of variation against the thickness to inner radius ratio. The value of initial stress P=0.1, 0.2 has been taken for cylinder-I and cylinder-II. From this figure it is observed that, frequency decreases with increase in ratio of thickness to inner radius. This discrepancy is due to the presence of magnetic field and initial stress within the solid portion.

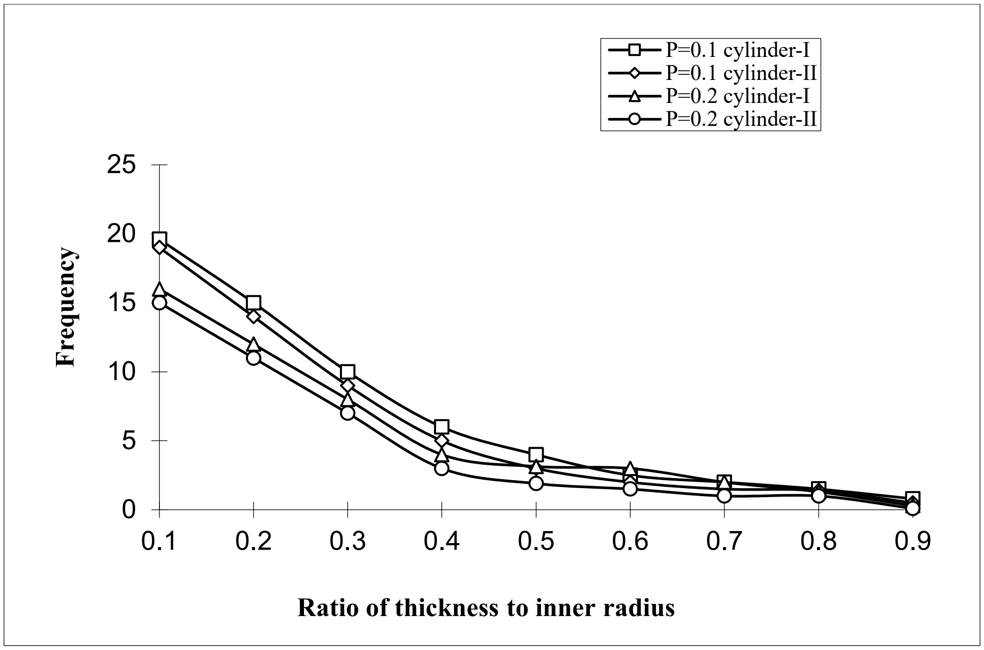

Fig:1 Variation of frequency against ratio of thickness to inner radius

CONCLUSIONS

It is observed that there is a significant effect of initial stress and magnetic field in the torsional vibrations in an anisotropic porous media. Numerical analyses were carried out to demonstrate the material anisotropy effect on stress, displacement, and pore pressure distributions in hollow cylinder. It is found that frequency decreases with increase in ratio of thickness to inner radius. The method presented in the paper is used in mechanics of geoengineering applications.

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